Performance of wireless receiver with an AFC loop in the presence of $k$-$\mu$ multipath fading and single CCI

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Abstract—In this paper the wireless communication system with the receiver consisting of an automatic frequency control (AFC) loop is considered. Proposed system is subjected to $k$-$\mu$ multipath fading environment and single interference. Performance measures such as average switching rate (ASR) and mean time to loss of lock (MTTL) are investigated. Furthermore, closed form expressions for ASR and MTTL are obtained. Numerical results are graphically presented in order to show the influence of Rician factor and fading severity parameter on ASR and MTTL.

Keywords—Automatic frequency control loop, average switching rate, co-channel interference, $k$-$\mu$ multipath fading, mean time to loss of lock.

I. INTRODUCTION

Wireless receiver with an automatic frequency control (AFC) loop is often integrated in wireless communication systems for the purpose to control the frequency of received signal [1]. Performances of an AFC in the presence of multipath fading and single co-channel interference (CCI) are well examined in [2-6]. It has been presented that AFC locks on the desired signal if the desired signal envelope is larger than interference envelope [2]. The degradation of the system performance of an AFC is caused when the AFC stops its tracking on the desired signal and locks on the CCI. This can happen due to the fact that amplitudes of desired signal and CCI depend on physical phenomena such as fading. Important performance measure which gives transitions from desired signal envelope to the CCI envelope and vice versa is average switching rate (ASR), already introduced in [2] and examined [2-6]. Mean time to loss of lock (MTTL) is the second performance measure also investigated [2-6], important to fully characterize system performances of an AFC. The MTTL gives the average time that an AFC stays locked on the desired signal of an AFC output.

The MTLL and ASR of an AFC in the presence of Rayleigh fading and single CCI in a closed form expressions are calculated in paper [3]. Furthermore, the MTLL and ASR of an AFC for the case of Rayleigh, Rician and Nakagami fading in the presence of a single CCI are examined in [4-5]. Case when desired signal and CCI are subjected to different fading channels are taken into account in paper [6], where the ASR and MTLL of an AFC of dissimilar fading distribution are calculated.

In this paper, ASR and MTTL of wireless communication system consisting of an AFC loop operating over $k$-$\mu$ multipath fading environment in the presence of single CCI also subjected to $k$-$\mu$ fading are evaluated. The $k$-$\mu$ distribution describes linear, line-of-sight (LOS) environments with two or more clusters. This distribution has recently been proposed [7]. Moreover, the $k$-$\mu$ distribution is general distribution, which means that for different $k$-$\mu$ fading parameters Nakagami-m, Rayleigh, and Rice fading distributions, as the special cases of $k$-$\mu$ distribution can be obtained. Therefore, in order to derive more general results $k$-$\mu$ multipath fading environment is considered. The numerical results are presented and discussed to show the influence of $k$-$\mu$ parameters on the system performances of an AFC.

II. ASR OF AN AFC

It has been stated that an AFC will lock on the signal with the larger amplitude between desired signal and the interferer under proposed conditions [2]. The desired signal envelope $x_1$ and interference envelope $x_2$ follow $k$-$\mu$ distribution[7]:

$$p_{x_1}(x_1) = \frac{2\mu_1(k_1 + 1)}{k_1^2} \frac{\mu_1 - 1}{2} e^{\frac{-\mu_1}{2} x_1^{\mu_1}}$$

$$\times I_{\mu_1 - 1} \left( 2\mu_1 \frac{k_1(k_1 + 1)}{\beta_1} e^{\frac{-\mu_1}{2} \frac{k_1}{\beta_1} x_1^2} \right) , i = 1, 2$$

(1)

where $k_1$ and $\mu_1$ are Rice factor and fading severity factor of the desired signal and interference, respectively and $\Omega_1$ is related to the local mean power of $x_1$. $I_{\nu}(\cdot)$ denotes the modified Bessel function of the first kind and order $\nu$ [10]. The Eq. (1) can be transformed by utilization of modified Bessel function of the first kind [9, eq. (03.02.06.0002.01)] as:

$$p_{x_2}(x_2) = \frac{2\mu_2(k_2 + 1)}{k_2^2} \frac{\mu_2 - 1}{2} \sum_{m=1}^{\infty} \left( k_2 \frac{1}{k_2} \right)^m e^{\frac{-\mu_2}{2} \frac{k_2}{\beta_2} x_2^2} I_{\nu}(\beta_2)^{-1}$$

$$\times \frac{1}{j_{\nu}(\beta_2)} x_2^{j_{\nu}(\beta_2) - 1} e^{-\frac{\mu_2}{2} \frac{k_2}{\beta_2} x_2^2} ,$$

(2)
where \( \Gamma(.) \) is gamma function.

It has already been stated that an AFC will lock on the signal with the larger amplitude. Accordingly, the difference between two \( k-\mu \) random variables is considered:

\[
x = x_1 - x_2
\]

The average switching rate of \( x \) is equal to the zero level crossing rate (LCR) in positive going plus negative going directions [9]:

\[
N = N_x(0) = \int_{-\infty}^{\infty} |x| f_{X,X}(0,\dot{x}) d\dot{x}
\]

The ASR of an AFC can be expressed as in [5]:

\[
N = f_x(0) \int_{-\infty}^{\infty} |x| \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2\pi}} d\dot{x} = f_x(0) \sqrt{\frac{2\pi}{\pi}}
\]

where \( f_x(0) \), according to [5] is equal to:

\[
f_x(0) = \int_{0}^{\infty} p_{x_1}(x)p_{x_2}(x) dx.
\]

Substituting Eq. (2) in Eq. (6), one has:

\[
f_x(0) = \frac{\mu_1 (k_1 + 1) \mu_1^{k_1} 2 \mu_2 (k_2 + 1) \mu_2^{k_2}}{k_1 \mu_1^{k_1+1} 2k_2 \mu_2^{k_2+1}} \times \sum_{j_1=1}^{\infty} \left( \frac{\mu_1^{(k_1+j_1+1)/2}}{\mu_1^{(k_1+1)/2}} \right) \frac{1}{j_1! \Gamma(1/2 + \mu_1)}
\]

\[
\times \sum_{j_2=1}^{\infty} \left( \frac{\mu_2^{(k_2+j_2+1)/2}}{\mu_2^{(k_2+1)/2}} \right) \frac{1}{j_2! \Gamma(1/2 + \mu_2)}
\]

\[
\times \frac{\alpha_1 \alpha_2 \mu_1^{\alpha_1+\alpha_2} \mu_2^{\alpha_1+\alpha_2} \Gamma(\alpha_1 + \alpha_2 + 1)}{\Gamma(\alpha_1 + \alpha_2) \Gamma(\alpha_1) \Gamma(\alpha_2)}
\]

\[
\times F(j_1 + j_2 + \mu_1 + \mu_2).
\]

III. \( \text{MTTL OF AN AFC} \)

The MTTL \( (T) \) of an AFC subjected to \( k-\mu \) multipath fading channel in the presence of interference can be derived using the formula [5]:

\[
T = \frac{2F}{N},
\]

where \( F \) is the probability that the \( x_1 \) is larger than \( x_2 \). According to [5], one has:

\[
F = P(x_1 > x_2)
\]

\[
= \int_{x_2}^{\infty} dx_1 \int_{0}^{\infty} p_{x_1}(x_1)p_{x_2}(x_2) dx_2.
\]

Substituting Eq. (2) in Eq. (7), the MTTL becomes:

\[
F = \frac{\mu_1 (k_1 + 1) \mu_1^{k_1+1} \mu_2^{k_2+1}}{k_1 \mu_1^{(k_1+1)/2} \mu_2^{(k_2+1)/2}} \times \frac{\alpha_1 \alpha_2 \mu_1^{\alpha_1+\alpha_2} \mu_2^{\alpha_1+\alpha_2} \Gamma(\alpha_1 + \alpha_2 + 1)}{\Gamma(\alpha_1 + \alpha_2) \Gamma(\alpha_1) \Gamma(\alpha_2)}
\]

\[
\times F(j_1 + j_2 + \mu_1 + \mu_2).
\]
where \( \gamma(n, x) \) is the Gamma function [10]. Applying the following transformation [10]:

\[
\gamma(n, x) = \frac{\mu^n}{n} e^{-\mu} F(1, 1 + n; x).
\]

where \( F(a, b; c; z) \) is the confluent hypergeometric function and using the integral [10]:

\[
\int_0^\infty t^{b-1} F(a, c; b; z) e^{-\mu} dt = \Gamma(b) s^{-b} F(a, b; c; z), |s| > |k|.
\]

where \( F(a, b; c; z) \) is Gauss hypergeometric function, one can obtained the closed form solution for MTLL of an AFC for the proposed model:

\[
F = \sum_{i=1}^{\infty} \left( \frac{k_i}{\mu_i} \right)^{\frac{1}{b^2}} \frac{\mu_i k_i + 1}{\mu_i k_i + 1} \frac{1}{j_i |\mu_i|^{1/2}}
\]

\[
\times \sum_{j=1}^{\infty} \left( \frac{k_j}{\mu_j} \right)^{\frac{1}{b^2}} \frac{\mu_j k_j + 1}{\mu_j k_j + 1} \frac{1}{j_j |\mu_j|^{1/2}}
\]

\[
\times \frac{\mu_1 (k_1 + 1)}{\mu_1 k_1 + 1} \frac{\mu_2 (k_2 + 1)}{\mu_2 k_2 + 1} \frac{1}{j_1 |\mu_1|^{1/2}}
\]

\[
\times \frac{\mu_2 (k_2 + 1)}{\mu_2 k_2 + 1} \frac{1}{j_2 |\mu_2|^{1/2}}
\]

\[
\times \Gamma \left( j_1 + j_2 + \mu_1 + \mu_2 \right)
\]

\[
\times F_1 \left( 1, j_1 + j_2 + \mu_1 + \mu_2 ; 1 + j_1 + j_2 + \mu_1 + \mu_2 ; \frac{\mu_1 (k_1 + 1)}{\mu_1 k_1 + 1} \frac{\mu_2 (k_2 + 1)}{\mu_2 k_2 + 1} \frac{1}{j_1 |\mu_1|^{1/2}} \right)
\]

IV. NUMERICAL RESULTS

Figs. 1 and 2, show the influence of different \( k-\mu \) fading parameters on ASR while figs. 3 and 4, show the influence of different \( k-\mu \) fading parameters on MTLL of an AFC.

In Fig 1, normalized ASR of an AFC versus the signal-to-interference ratio (SIR) for the constant fading severity parameters of the desired branch, \( \mu_1 \) and CCI branch, \( \mu_2 \) and different Rice factors of desired branch, \( k_1 \) and CCI branch, \( k_2 \) is presented. The ASR is normalized to \( f_m \) and the SIR is given as \( SIR = 10 \log_{10} \frac{\Omega_1}{\Omega_2} \). By increasing the Rice factor in both branches for constant values of \( \mu_1 \) and \( \mu_2 \) the performance improves in the sense that the ASR decreases. In the boundary case that the Rice factors tend to infinity, the AFC locks on the signal with more power and there will be no switching.

In Fig 2, normalized ASR of an AFC versus SIR for different parameters of \( \mu_1 \) and \( \mu_2 \) and constant parameter \( k_1 \) and \( k_2 \) is presented. By increasing the \( \mu_1 \) and \( \mu_2 \) in both branches, the performance improves in the sense that the ASR decreases. Moreover, this figure shows that for a constant fading severity factor in the CCI branch, increasing the \( \mu_1 \) causes the ASR to decrease, which contributes to the improvement in the performance for larger values of SIR, while it does not have a significant effect on the ASR for small values of SIR.

In Fig 3, shows the MTLL multiplied by \( f_m \) of an AFC for \( k-\mu \) fading channel for the case when \( \mu_1 \) and \( \mu_2 \) are constant and \( k_1 \) and \( k_2 \) take different values. Similarly, by increasing \( k_1 \) and \( k_2 \) the MTLL increases for larger values of SIR. It can be seen that if the channel of the desired signal has larger values of \( k_1 \) (stronger LOS component), the performance of the AFC improves in the sense that the MTLL of the AFC increases for the same values of SIR.
In the Presence of Interference, one of the receiver, one also need to find the MTLL to fully evaluate AFC performance.

Note that for the special case when $k=0$, $k$-$\mu$ distribution approximates Nakagami-m distribution. Furthermore, $k$-$\mu$ distribution approximates Rice distribution for $\mu=1$ while Rayleigh distribution is derived for $k=0$ and $\mu=1$. Moreover, the obtained results are in accordance with previously published references [4-5].

V. CONCLUSION

The closed form expressions for the performance measure such as ASR and the MTLL of an AFC in the presence of interference over $k$-$\mu$ multipath fading are derived. Numerical results are presented and discussed to show the effect of the $k$-$\mu$ fading parameters on the performance of an AFC. For the first time, $k$-$\mu$ multipath fading is considered, to show the influence of more general fading environment on the performance of an AFC.

REFERENCES