Calculation of Magnetic Field and Supercurrent Distributions of Type-II Superconductors in the Mixed State using Modified London Model

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Abstract—**Calculation of magnetic field and supercurrent distribution has been performed, using modified London model. We have written a simulation in MATLAB that calculates a mixed state of type-II superconductors. Starting point of the calculations was magnetic field distribution. Supercurrent distribution was obtained from the magnetic field distribution, according to the Ampere's law. It has been shown that it is possible to extract the vortex-core size from the profile of supercurrent distribution. Dependence of the vortex-core size upon the cutoff parameter is provided. Two basic cases of the vortex lattice (VL) have been used – square and triangular lattice.**

Key words-superconductors; type-II; mixed state; London model; vortex lattice (VL); vortex-core size; simulation; MATLAB

I. INTRODUCTION

Superconductivity (SC) is a physical phenomenon characterized by vanishing of the electric resistivity in different materials and alloys when they are cooled down below a certain temperature, known as the critical temperature (T_c). Beside the stepwise decrease of the specific electric resistivity to zero, it was observed that a superconductor pushes out the magnetic flux, when placed in an external magnetic field. This appearance is called Meissner effect. The explanation is that *surface supercurrents* are induced and their magnetic field cancels out the external magnetic field. Inside the superconductors, electric field is $E = \rho J = 0$ (since $\rho = 0$), therefore only surface currents can exist. Under the condition $T < T_c$, there is a surface supercurrent whose magnetic field completely cancels out external magnetic field, under the condition that external magnetic field is smaller than the *critical field* ($H < H_c$). The value of critical field depends on the temperature. It decreases from its maximum value $H_c(0)$ to the zero value at critical temperature ($H_c(T_c) = 0$). When one places already cooled superconductor ($T < T_c$) in a magnetic field bigger than critical field at that temperature ($H > Hc(T)$), material loses SC characteristics and Meissner effect does not happen. In principle, one can define superconductivity as a characteristic behaviour of a given material. SC exists in a certain space of parameters, like H(T) defined by the values of magnetic field H and temperature T.

There are two possible cases, depending exclusively on the kind of the SC material. In type-I superconductors, when $H = Hc(T)$, whole sample returns to a *normal state*, when magnetic induction B completely penetrates into the sample. In type-II superconductors, when $H < H_{c1}(T)$, where H_{c1} is the lower critical field, magnetic induction does not penetrate into the sample and this is called – *superconducting state*. When $H > H_{c2}(T)$, where H_{c2} is the upper critical field, sample returns to a normal state, with full penetration of the magnetic induction. Under the condition $H_{c1}(T) < H < H_{c2}(T)$, there is a partial penetration of magnetic induction into the sample. Regular microscopic structure with alternate placements of normal and superconducting areas settles, this is known as the *mixed state*. In the mixed state magnetic induction enters the sample partly, as magnetic vortex lines. Alexei Abrikosov won a Nobel prize in physics for his prediction about the mixed state. He predicted that some materials could preserve SC in strong magnetic fields by allowing the external magnetic field to penetrate the sample as a periodic arrangement of quantized flux lines, this is a vortex lattice (VL) [1].

The basis of Abrikosov's model of the mixed state is phenomenological Ginzburg-Landau (GL) theory. The structure of the VL is, within the GL theory, characterized by the two fundamental length scales. Those are *magnetic field penetration depth* - λ and *coherence length* - ξ. The first phenomenological theory of superconductivity was London theory. A major triumph of the equations of this theory is their ability to explain the Meissner effect. Their equation shows that magnetic induction in the superconductor decays exponentially with the distance from the surface. It means that a superconductor in Meissner state is not ideal because magnetic induction penetrates into the sample on the length scale of λ – London penetration depth. One way of explaining the other parameter, coherence length, is that it defines the length scale over which the transition between normal and SC areas in the mixed state happens. There are different experimental techniques that investigate the mixed state of type-II superconductors. One of the most powerful among them is the muon spin rotation technique.

Muon spin rotation is an experimental technique used to measure local magnetic fields inside the sample. It can probe magnetic induction in the bulk of a superconductor in a vortex lattice state [3]. Those measurements can determine the vortex-core size.

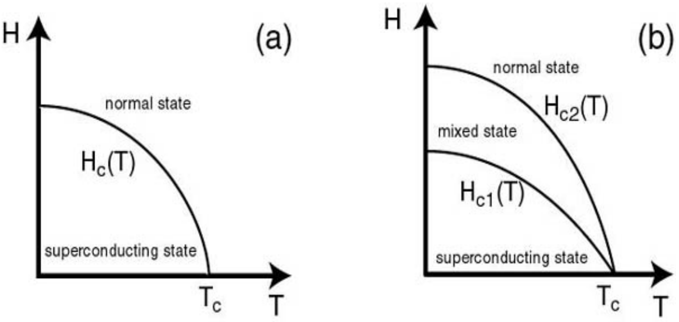

Figure 1. $H_c(T)$ characteristic for type-I and type-II superconductors

II. LONDON MODEL

There are different models for the description of magnetic field distribution of the mixed state in type-II superconductors. The aim of those models is to extract data from muon spin rotation experiments. The difference between the models is in their ability to take into account various effects which can be measured in the experiments. In phenomenological London model, magnetic field for a perfect flux-line lattice (FLL) [3] is

$$\vec{h}_L(\vec{r}) = \vec{e}_z \frac{\Phi_0}{S} \sum_{\vec{G}} \frac{e^{i\vec{G}\cdot\vec{r}}}{1+\lambda^2 G^2} \tag{1}$$

where the sum goes over the reciprocal lattice vectors $\vec{G}$ of the FLL. Here Φ_0 is a flux quantum with its normalized value of 2π. Parameter S is the area of the vortex lattice unit cell. Vector $\vec{r}$ is the radius vector in the *xy* plane, it determines the position of the point in which magnetic field is calculated. Parameter λ is the penetration depth in the Meissner state. In reference [5] the temperature dependence of λ is given in the Eq. 2. However, we are dealing with a simplified model that does not account for the temperature dependence, so parameter λ has a fixed normalized value.

III. MODIFIED LONDON MODEL

London model used for the analysis of the experimental data does not account for the spatial dependence of the order parameter and it breaks down at distances on the order of coherence length from the vortex core center, B(r) diverges. To correct this, the sum over $\vec{G}$ can be truncated by multiplying each term in Eq. 1 by a cutoff function F(G), where G is the module of $\vec{G}$. One important fact is that the adequate form of F(G) depends on the precise spatial dependence of the order parameter in the vortex core, which in general depends on temperature and magnetic field. Modified London model has the analytical expression for F(G) in the form of a smooth Gaussian cutoff factor [3]

$$F(G) = \exp(-\alpha G^2 {\xi_h}^2) \tag{2}$$

Magnetic field distribution in modified London model is given as

$$\vec{h}_L(\vec{r}) = \vec{e}_z \frac{\Phi_0}{S} \sum_{\vec{G}} \exp(-\alpha G^2 {\xi_h}^2) \frac{e^{i\vec{G}\cdot\vec{r}}}{1+\lambda^2 G^2} \tag{3}$$

Brandt determined the values of α by solving Ginzburg-Landau (GL) equations [5]. Furthermore, Laiho et al. have shown that α is temperature dependent [2]. They have determined the temperature and magnetic field dependences of the cutoff function used in the modified London model. These calculations show that the London model with the proper cutoff function provides a reasonable description of the magnetic field distribution of the FLL in type-II superconductors.

IV. VORTEX LATTICE CONFIGURATION

Before the calculation of magnetic field, shape of the vortex lattice has to be fixed. The shape of the VL is determined by the unit vectors [4]

$$\begin{aligned} \vec{r}_1 &= (a,0,0) \\ \vec{r}_2 &= (a\cos\theta_L, a\sin\theta_L, 0) \\ \vec{r}_3 &= (0,0,1) \end{aligned} \tag{4}$$

where a is the lattice constant

$$a = \sqrt{\frac{\Phi_0}{H}}$$

The range for H is $0 < H < 1$. The lattice characteristic angle θ_L is the angle between $\vec{r}_1$ and $\vec{r}_2$. Two basic shapes are considered in this paper: square lattice ($\theta_L = 90°$) and triangular lattice ($\theta_L = 60°$). By introducing reciprocal unit vectors

$$\begin{aligned} \vec{k}_1 &= 2\pi \frac{\vec{r}_2 \times \vec{r}_3}{S} \\ \vec{k}_2 &= 2\pi \frac{\vec{r}_3 \times \vec{r}_1}{S} \end{aligned} \tag{5}$$

that define reciprocal lattice vector $\vec{G}$, where n and m are integers

$$\vec{G}(n,m) = -n\vec{k}_1 + m\vec{k}_2 \tag{6}$$

Area of the vortex unit cell is calculated as

$$S = \vec{r}_1 \cdot (\vec{r}_2 \times \vec{r}_3) \tag{7}$$

The sum over reciprocal lattice vector $\vec{G}$ is symmetric.

It means that integers (n,m) take the values from the interval

$$-l \leq \{n, m\} \leq l \tag{8}$$

Value of l is determined under the cutoff condition [5],

$$\left|\vec{G}_{\max}\right| \approx \frac{2\pi}{\xi_h} \tag{9}$$

From the Eq. 6 we obtain the value of G

$$\left|\vec{G}\right| = \sqrt{(mk_{2x} - nk_{1x})^2 + (mk_{2y} - nk_{1y})^2} \tag{10}$$

A. Square lattice

In the case of square lattice, Eq. 10 simplifies to

$$\left|\vec{G}\right| = \sqrt{(nk_{1x})^2 + (mk_{2y})^2} \tag{11}$$

Since $k_{1x} = k_{2y}$ and $n_{\max} = m_{\max} = l$, we obtain

$$\left|\vec{G}\right|_{\max} = l\sqrt{2}k_{1x} \tag{12}$$

Combining Eq. 9 and Eq. 12 gives us

$$l = \frac{2\pi}{\sqrt{2}\xi_h k_{1x}} \tag{13}$$

According to Eq. 5, $k_{1x} = \frac{2\pi}{a}$, therefore Eq. 13 becomes

$$l = \frac{a}{\sqrt{2}\xi_h} \tag{14}$$

B. Triangular lattice

In the case of triangular lattice, Eq. 10 simplifies to

$$\left|\vec{G}\right| = \sqrt{(nk_{1x})^2 + (mk_{2y} - nk_{1y})^2} \tag{15}$$

Maximum value of G is

$$\left|\vec{G}\right|_{\max} = l\sqrt{(k_{1x})^2 + (k_{2y} + |k_{1y}|)^2} \tag{16}$$

Combining Eq. 9 and Eq. 16, gives us

$$l = \frac{2\pi}{\xi_h \sqrt{(k_{1x})^2 + (k_{2y} + |k_{1y}|)^2}} \tag{17}$$

According to Eq. 5, $k_{1x} = \frac{2\pi}{a}, k_{1y} = -\frac{2\pi}{a\sqrt{3}} k_{2y} = \frac{4\pi}{a\sqrt{3}}$

$$l = \frac{\sqrt{3}a}{2\xi_h} \tag{18}$$

V. CURRENT DISTRIBUTION

Magnetic field distribution over a chosen *xy* mesh has been calculated, according to Eq. 3. Next parameter of a type-II superconductor we want to calculate is the vortex-core size. One of possible ways for defining a vortex-core size is provided in [6]. It is defined from the supercurrent density near the vortex center. At the vortex center supercurrent density is equal to zero, since that part of the superconductive material is not in SC state. The absolute value of supercurrent density reaches its maximum value at the distance R_0 from the vortex center. This distance is the vortex-core size. When there is an analytical expression for the magnetic field, current distribution is easily calculated. According to the Ampere's law (IV Maxwell's equation)

$$\vec{J}(\vec{r}) = \vec{\nabla} \times \vec{h}(\vec{r}) \tag{19}$$

Magnetic field points along the z-direction. By solving the determinant that corresponds to the vector product from Eq. 19, the expression for current distribution is obtained

$$\vec{J}(\vec{r}) = \vec{e}_x \frac{\partial h(\vec{r})}{\partial y} - \vec{e}_y \frac{\partial h(\vec{r})}{\partial x} \tag{20}$$

Let us calculate the *x* and *y* components of the current:

$$\frac{\partial h(\vec{r})}{\partial \varepsilon} = i\frac{\Phi_0}{S}\sum_{\vec{G}}\frac{F(G)e^{i\vec{G}\cdot\vec{r}}}{1+\lambda^2G^2}G_\varepsilon, \varepsilon = \{x, y\} \quad (21)$$

Intensity of the current in a given point of the *xy* mesh is

$$J(\vec{r}) = \sqrt{J_x(\vec{r})^2 + J_y(\vec{r})^2} \quad (22)$$

VI. RESULTS

Magnetic field and current distribution of modified London mode have been calculated, in the cases of square and triangular vortex lattices. We show the results for $\xi_h = 0.18$. Furthermore, the values of vortex-core size have been extracted from the profile of the current distribution. Dependence of vortex-core size upon the cutoff parameter has been found.

A. Square lattice

In Fig. 2 and Fig. 3 top view and side view of magnetic field distribution in square lattice case are shown, respectively.

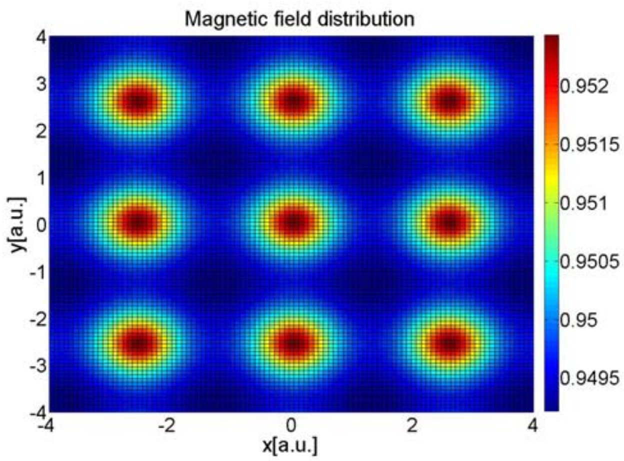

Figure 2. Top view of magnetic field distribution (square lattice)

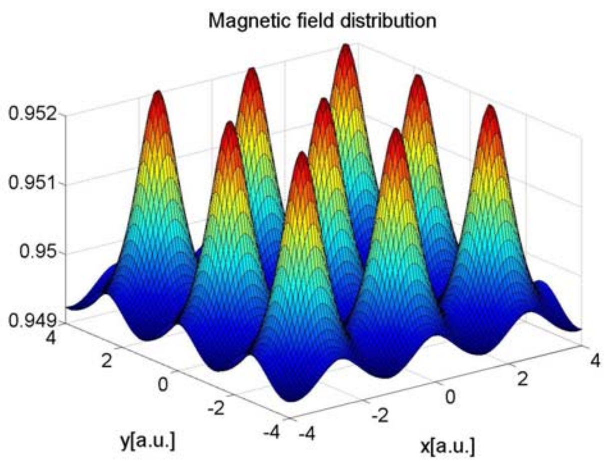

Figure 3. Side view of magnetic field distribution (square lattice)

It is confirmed that the lattice constant extracted from Fig. 2 is equal to the lattice constant given in the simulation. In Fig. 3 we can observe that magnetic field has maximum value in the core center and symmetrically drops down around the core center. In Fig. 4 and Fig. 5 top view and side view of current distribution are shown.

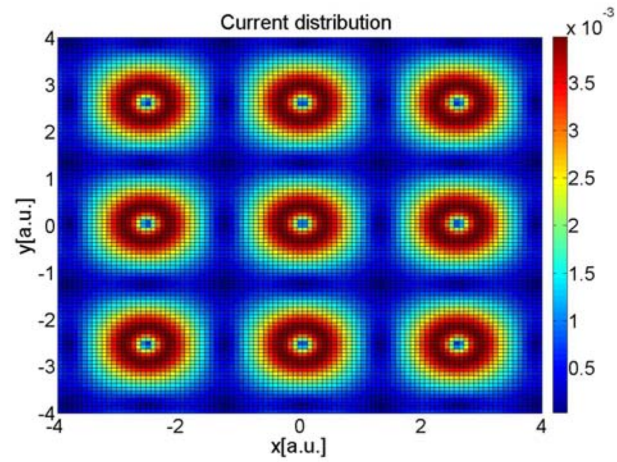

Figure 4. Top view of current distribution (square lattice)

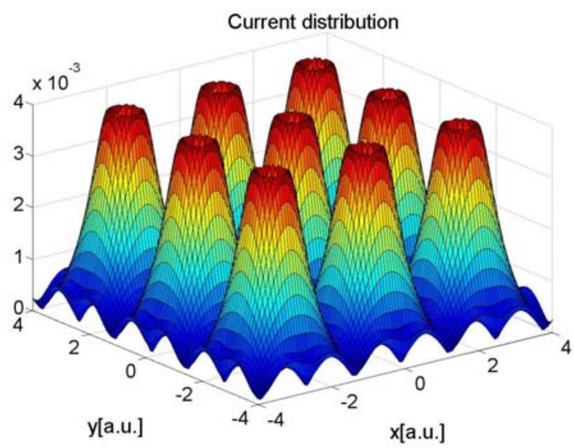

Figure 5. Side view of current distribution (square lattice)

One can notice the square lattice in Fig. 4, with the proper lattice constant. In Fig. 5 one can observe that current has minimum value in the core center and symmetrically rises up around the core center, in "volcano-like" shape.

B. Triangular lattice

Fig. 6 and Fig. 7 top view and side view of magnetic field distribution in triangular lattice case are shown, respectively. One can notice the triangular lattice in Fig. 6, with the lattice constant equal to the lattice constant calculated in the simulation. In Fig. 7 one can observe that magnetic field has maximum value in the core center and symmetrically drops down around the core center.

In Fig. 8 and Fig. 9 top and side view of current distribution are presented. Similar to the square lattice case, current has minimum value in the core center, and rises up symmetrically around the core center.

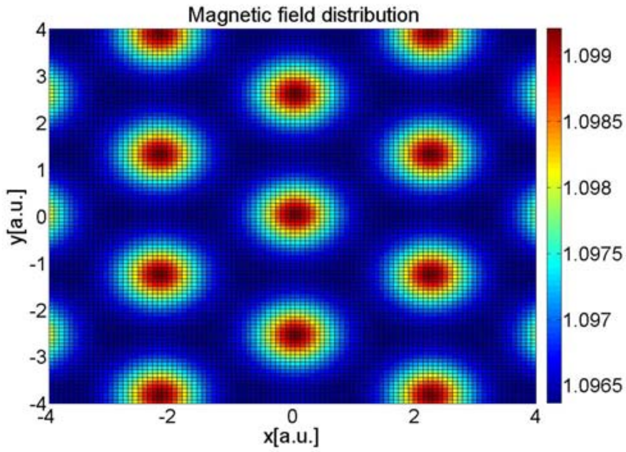

Figure 6. Top view of magnetic field distribution (triangular lattice)

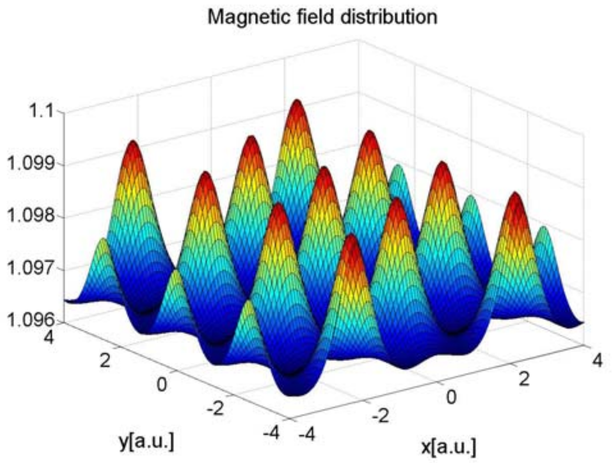

Figure 7. Side view of magnetic field distribution (triangular lattice)

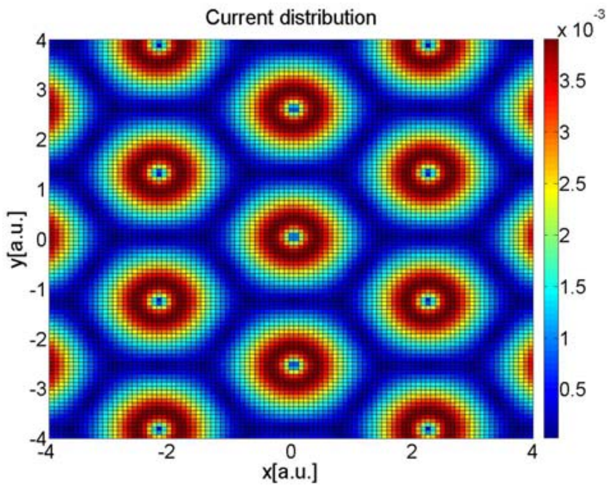

Figure 8. Top view of current distribution (triangular lattice)

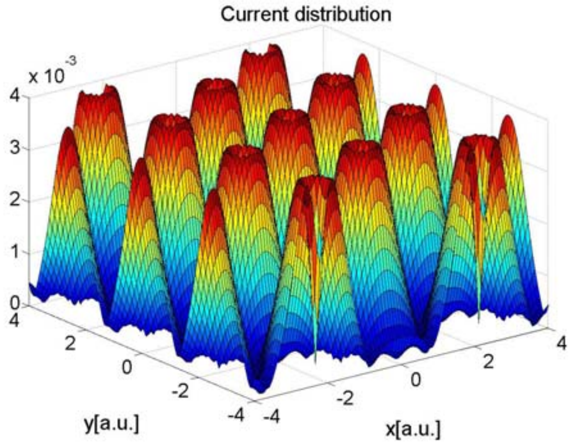

Figure 9. Side view of current distribution (triangular lattice)

Now when we have the current distribution, next step is to plot its profile and extract the vortex-core size. As stated in [6] (Fig. 6 (b)) vortex core size is the distance from the vortex centre at which supercurrent density has maximum value. In Fig. 10 profile of the supercurrent distribution is presented, for $\xi_h = 0.18$.

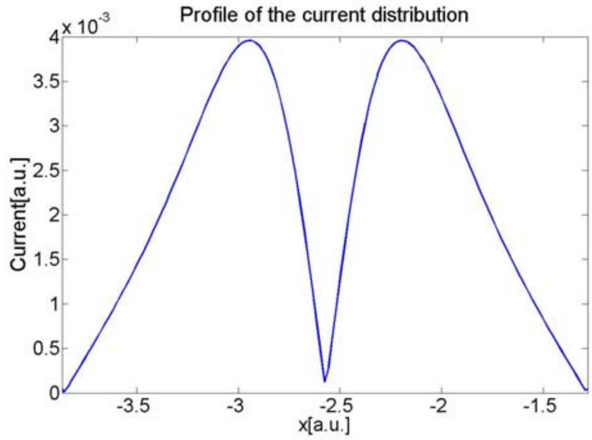

Figure 10. Profile of the current distribution ($\xi_h = 0.18$)

The vortex-core size does not depend on the lattice shape, for one value of ξ_h it is the same for both lattices. The profile along *x*-axis is presented, it is the same along *y*-axis, due to symmetry. The *xy* mesh includes three vortex cores along both axes. This plot has been obtained by observing one arbitrary chosen vortex core. In Fig. 11 the dependence of vortex-core size on the cutoff parameter is shown.

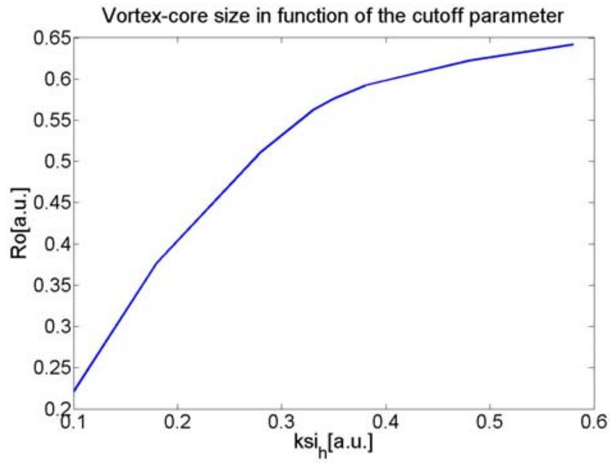

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Figure 11. Vortex-core size in function of the cutoff parameter

VII. CONCLUSION

Magnetic field and supercurrent distribution have been calculated according to the modified London model. The results have met expectations, meaning that the lattice shape with proper lattice constant has been obtained. Distributions correspond to the mixed state of a type-II superconductor. Vortex-core size has been obtained from the profile of the supercurrent distribution. Plot of the vortex-core size versus cutoff parameter shows that they are on the same order of magnitude and that an increase of the cutoff parameter causes the increase of the vortex-core size. Future work may include the analysis and application of the models that include temperature dependance, which will allow comparison and matching of the calculations with experimental results.

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